Evaluation of Grasp Stiffness in Underactuated Compliant Hands

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***Abstract*— Underactuation represents a solution to reduce the number of Degrees of Freddom (DoF) of robotic hands. Altough reducing the number of DoFs in general limits the ability to perform many and different types of grasp, the use of springs in the structure improves the compliance of the grasp and mitigates the loss of generality due to the reduction of DoFs in the mechanical structure. The use of active and passive elastic elements improves robustness of the whole grasp. In this paper we evaluate the grasp stiffness which depends on the structural compliance of hand links and joints and of the contacts, and on the gains of impedance controllers. A quasi-static model of the grasp for underactuated hands is presented and used to explicitly compute the grasp compliance.**

I. Introduction

One of the main challanges in robotic hand design consists of simplifying the structure of the hand reducing the number of actuators while keeping good levels of robustness and dexterity. This can be obtained reducing the number of Degrees of Freedom (DoF) by coupling joint motions using for instance tendons [1] or fixed mechanical couplings [2]. Another possible solution towards simplification of hands is to introduce passive compliance at the joints, which allows to improve adaptability properties and gaining robustness with respect to uncertainties [3]. Generally speaking, a mechanism is defined underactuated when it has more DoF than actuators [4]. Decreasing the number of actuators may deteriorate manipulability and controllability performances of the hand, as discussed in [5] where the authors studied robotic hands with joints coupled through a compliant system, inspired by the synergy organization of human hand motion [6].

In order to maintain the versatility properties while simplifying robotic hand structure through underactuation and passive joints, theoretical tools that allows the design and optimization of hand parameters are needed. In [7] the kinetostatic analysis of underactuated robotic hands is presented. The force distribution during grasping operation is analysed and tools for the investigation of grasp stability are provided. The form closure properties have been extended to underactuated hands in [8]. The manipulability analysis of hands with passive joints has been studied in [9]. In [10] dexterous manipulation properties with underactuated elastic hands is discussed. In [11] the authors discusses the problem of force isotropy in underactuated hands, this property guarantees a uniform distribution of forces over the grasped object and prevents object damages due to force unbalances.

One important parameter that has to be considered in the design of robotic hands is the stiffness. This parameter is particularly significant in grasp operations, when the hand has to apply to the object a force and impose a motion [12]. In general, stiffness analysis evaluates the resistance of the robot system to the deformation caused by an external load variation applied on the end effector. In grasping, stiffness refers to the abality of the grasped object to resist to external load variations.

In robotics, traditionally researchers distinguish between mechanical stiffness, or passive compliance, that is caused by the structural deformation of robot components when the system is subject to a load, and the virtual elasticity of actuators, that can be adjusted by acting on the control parameters [13].

Different methods are available for the evaluation and representation of a manipulator mechanical stiffness. In this paper we will refer to the so-called Virtual Joint Method [14] , that substantially represents the robot as a multi-body system composed of rigid bodies, where the links are connected by compliant joints. This technique is more approximated than other methods (e.g. Finite Element Methods, FEM), but on the other hand, due to its simplicity, it is more suitable to grasp the main properties which infulence the designof the overall dynamic properties of the robotic hand.

Furthermore, conventional stiffness analysis usually considers the unloaded case. In this work, we consider the loaded case which considers the so-called *geometric stiffness terms* as a function of the hand configuration variation and thus depending on the second derivative of kinematic relationships [14], [15], [16].

Compliance is crucial to allow underactuated hands to adapt to different objects and grasping tasks and have to be properly designed [17].

In [5] the controllability problems that arises when the hand is controlled with a reduced number of actuators was analysed. Hand joints were considered coupled according to a compliant model defined as *soft synergies* [18]. This paper integrates the quasi–static grasp model proposed in [5] with the treatment of underactuated hands, which include joints with passive elastic elements, as the one schematically represented in Fig. 1 which is a planar underactuated robotic hand where the proximal joints are actuated by two tendons while the distal ones have a passive elastic spring.

This paper extends the model introducing the possibility

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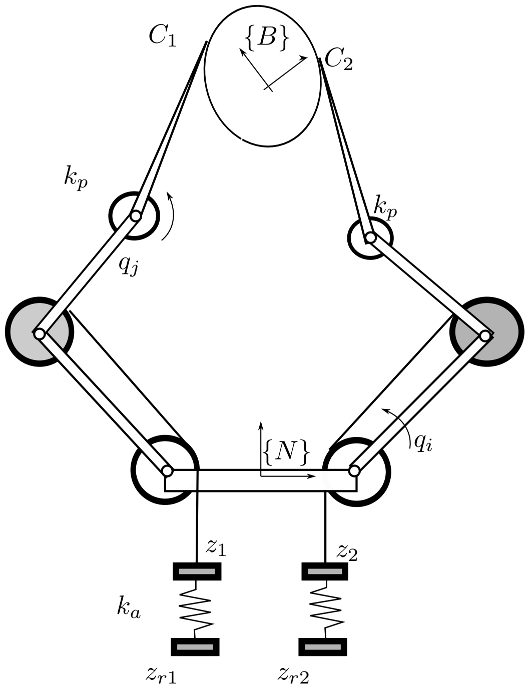

Fig. 1: An underactuated hand with passive joints, inspired from [10].

of havinf some uncontrolled, or passive joints in the mechanical structure of the robotic hand. The main equations that describes grasp statics and kinematics are summarized and in particular, the possibility that not all the hand DoF are actuated is considered. Starting from an initial reference equilibrium configuration, a small perturbation is imposed to the system, consisting of a variation of the actuator reference configurations and/or a variation of the external load applied to the object. For the perturbed condition the equilibrium equations are evaluated, using a linearized model. The obtained linear system that describes the perturbed hand configuration can be solved. In particular, in this paper we focused on the evaluation and analysis of grasp stiffness. The analytic expression of grasp compliance is computed showing dependency on hand parameters. In the final part of the paper, grasp stiffness is computed for a simple planar gripper considering two different actuation types.

II. Stiffness in Grasping with underactuated hands

A. Modeling the grasp

The stiffness of a grasp is defined as the linear relationship between a variation of the wrench applied on an object and the resulting motion

$$\Delta w = K \Delta u \tag{1}$$

where $w \in \Re^6$ is the wrench applied to the grasped object, $u \in \Re^6$ is a vector describing the object frame configuration $\{B\}$ with respect to an inertial frame $\{N\}$ and the prefix Δ indicates that we are considering a *small* variation of grasp configuration with respect to a reference equilibrium condition. The smallness of the variation allows the *linearization* of the kinematic and equilibrium relationships that characterize grasp configuration. Grasp stiffness matrix therefore $K \in \Re^{6\times 6}$ describes force/motion relationships in the object space [12].

Let n_c be the number of contact points. Let $\{C_i^h\}$ be the reference frame on the i-th contact point, connected to the hand, and $\{C_i^o\}$ the reference frame on the i-th contact point, connected to the object. To model grasp forces between the object and the hand, for each contact point i, we introduce the contact force $\lambda_i \in FC_i$, where $FC_i \subset \Re^{l_i}$ represents the friction cone and the value of l_i depends on the type of contact [19]. Introducing the above definition into the static equilibrium equation of the grasped object, we obtain

$$w + G\lambda = 0 \tag{2}$$

where $\lambda = [\lambda_1^{\mathrm{T}}, \cdots, \lambda_{n_c}^{\mathrm{T}}]^{\mathrm{T}} \in \Re^{n_l}$ is a vector containing all the contact forces, and $G \in \Re^{6\times n_l}$ is the *Grasp matrix* [19] Let $c^o \in \Re^{n_l}$ collect the constrained components of contact frame displacements for all the contact points, its variation can be evaluated as [20]

$$\Delta c^o = G^{\mathrm{T}} \Delta u \tag{3}$$

B. Modelling the underactuated hand

Let $q \in \Re^{n_q}$ be a vector that describes hand configuration. Typically the elements of q vector represent hand joint displacement. The components of contact point displacements on the hand constrained by the contact model, expressed with respect to $\{B\}$ reference frame, can be evaluated as

$$\Delta c^h = J \Delta q \tag{4}$$

in which $J \in \Re^{n_l \times n_q}$ represents the *hand Jacobian matrix* [19].

It is worth to observe that in equations (2) and (4) both the Grasp Matrix and Hand Jacobian are expressed with respect to the *object reference frame*, and by neglecting rolling between the fingers and the object in the contact points, G results to be constant, while $J(q, u)$, in general, depends on both hand and object configurations.

In a static equilibrium condition, the contact forces that the hand exchanges with the object are balanced by the joint torques τ

$$\tau - J^{\mathrm{T}} \lambda = 0 \tag{5}$$

In this paper we propose a quasi static model that can be applied both to underactuated hands [3] and to fixed motion coupled hands [21].

Firstly we suppose that the hand joint *reference* configuration q_r can be defined using a number of inputs whose dimension is lower than the number of hand joints, in other terms we assume that the relative motions of hand joints are somehow constrained. So, we can define a vector $z \in \Re^{n_z}$ of *Lagrangian variables*, whose dimension is equal to the number of hand DoFs [4], [8]. In the literature, approaches where the actual joint variables is a linear combination or a function of such Lagrangian variables have been presented [22], [23]. In these papers the joint displacement aggregation corresponds to a reduced dimension representation of hand movements according to a *compliant* model of joint torques, as introduced in [18].

Secondly, we suppose that the Lagrangian coordinates can be divided as $z = [z_a^{\mathrm{T}} \;\; z_p^{\mathrm{T}}]^{\mathrm{T}}$, with dimension $n_z = n_{za} + n_{zp}$ where $z_a \in \Re^{n_{za}}$ represents the *active* or *controlled* input variables, corresponding, for instance, to the actuators, while the remaining variables $z_p \in \Re^{n_{zp}}$ represents the uncontrolled or *passive* coordinates of the system.

The kinematic analysis of the mechanism allows expressing the *reference values* of the joint variables q_r, i.e. the values of the joint variables that would be obtained if the hand structure was perfectly stiff or if there were no external loads, as a function of the Lagrangian coordinates z

$$q_r = f_z(z) \tag{6}$$

where $f_z : \Re^{n_z} \rightarrow \Re^{n_q}$ represents the generally non–linear kinematic relationship.

This relationship can be linearized in order to express the joint displacement variation as a function of the Lagrangian coordinate variation:

$$\Delta q_r = S \Delta z \tag{7}$$

where $S = \frac{\partial f_z}{\partial z}|_0 \in \Re^{n_q \times n_z}$. In synergy actuated hands, as those presented in [5], matrix S corresponds to the *synergy matrix*, in [21] it is referred to as eigengrasp matrix. It is worth to note that, with respect to the existing literature, in this case matrix S includes also the terms that depends on passive Lagrangian variables and in general it depends on hand configuration, since f_z is not linear.

The forces and torques applied to the hand by the contacts with the object can be represented in the Lagrangian variable space as

$$\sigma = S^{\mathrm{T}} \tau \tag{8}$$

where $\sigma \in \Re^{n_z}$ represents the forces applied by the contact to the hand expressed in the Lagrangian variable space. In an equilibrium configuration, these forces will be balanced by the actuators actions and by the forces that arise when passive elements like springs are deformed from their reference configuration. Recalling the partition of Lagrange variables in active and passive variables, z_a and z_b respectively, the generalized forces σ can be either partitioned as

$$\sigma_a = S_a^{\mathrm{T}} \tau \qquad \sigma_p = S_p^{\mathrm{T}} \tau$$

representing the contribution to hand equilibrium of the forces generated by the actuators σ_a and of the passive elements σ_p.

C. Modeling grasp compliance

In case of a low number of DoFs like in undactuated hands, it is very likely that the grasp results to be defective, *i.e.* with a non trivial $\ker(J^{\mathrm{T}})$. It is worth underlying that this typically happens also in power grasps [24] where the hand envelops the object establishing contacts even with inner limbs. In this case the hand Jacobian is a tall matrix with a non trivial nullspace of its transpose. If the system is very defective, it is very likely that the grasp results to be hyperstatic and consequently we need to use a compliant model of contact interaction to solve the force distribution problem in grasping [19]. We introduce a set of *springs* placed between the fingers and the object at the contact points: a contact force variation $\Delta\lambda$ from an initial equilibrium configuration can be expressed as

$$\Delta\lambda = K_c(\Delta c^h - \Delta c^o) = K_c(J\Delta q - G^{\mathrm{T}}\Delta u) \tag{9}$$

where $K_c \in \Re^{n_l \times n_l}$ is the contact compliance matrix which is symmetric and positive definite. Of course contact compliance is necessary to be included in the model not only in hyperstatic grasps but also in grasps with compliant contacts typically obtained covering the fingers with rubber or other soft material or grasping compliant objects.

Another source of compliance in robotic hands is the structural stiffness of the links and the controllable servo compliance of the joints [12] which often have the same order of magnitude of the contact stiffness and thus cannot be disregarded [25]. In this paper we model the hand structure compliance and the servo compliance at the joints with this relationship

$$\Delta q_r - \Delta q = C_q \Delta\tau \tag{10}$$

Note that the inverse of compliance matrix $K_q = C_q^{-1} \in \Re^{n_q \times n_q}$ is defined as the hand stiffness matrix.

D. Soft synergies with passive joints

Also for the Lagrangian variables, we assume a compliant model. When the idea of hand synergies was initially applied to robotic hands, a rigid coupling between configuration variables and synergies was considered [21], that can be substantially represented as fixed motion coupled hands. This type of modeling approach presented some problems in the grasp analysis and to solve these problems a *softly underactuated model* was proposed in [18] where the synergies reference values z_{ra} are directly controlled by the synergistic actuation, while their actual values depends on the system stiffness and on the applied load.

In the soft synergy model proposed in q[18], the presence of passive joints in the hand structure was not explicitly taken into account, i.e. they assumed that $n_{za} = n_z$ and thus $n_{zp} = 0$. In this paper we extend the soft synergy model to include passive joints. For the active Lagrange variables, let us assume that the actuators are closed loop controlled in position, so that the input variable that the user control is the reference value of the actuator position, z_{ar}. The generalized actuation forces are then proportional to the difference between the reference and actual actuator position

$$\Delta\sigma_a = K_{za}(\Delta z_{ra} - \Delta z_a) \tag{11}$$

where $K_{za} \in \Re^{n_{za} \times n_{za}}$ is a symmetric and positive definite matrix modeling the actuator stiffness. Matrix K_{za} includes both the compliance of the actuation transmission elements (e.g. tendon elasticity), and the gain of actuator control systems. Also for the passive variables we assume a compliance model, then the Lagrangian forces corresponding to the passive joints are

$$\Delta\sigma_p = -K_{zp}\Delta z_p \tag{12}$$

where $K_{zp} \in \Re^{n_{zp}\times n_{zp}}$ is a symmetric and positive definite matrix. Combining eq. (11) and (12) we obtain

$$\Delta\sigma = \begin{bmatrix} \Delta\sigma_a \\ \Delta\sigma_p \end{bmatrix} = K_z(\Delta z_r - \Delta z) \tag{13}$$

where $K_z = \begin{bmatrix} K_{za}, & 0 \\ 0 & K_{zp} \end{bmatrix}$, $\Delta z_r = \begin{bmatrix} \Delta z_{ar} \\ 0 \end{bmatrix}$, $\Delta z = \begin{bmatrix} \Delta z_a \\ \Delta z_p \end{bmatrix}$.

III. Grasp analysis in underactuated compliant hands

Starting from the reference configuration, indicated with index $_0$ to perform the grasp analysis, consider a small variation of inputs. The grasp is here considered as a MIMO system where the inputs are the active Lagrangian variable reference values z_{ar} and the object wrench w, representing in general the interaction between the object and the environment. Note that in our analysis the reference variables z_{ar} are controllable while the external wrench applied to the object w is not controllable. We disregard inertia effects and assume to work under quasi–static conditions. This is typically the case when the applied variations to the inputs sufficiently slow and if the mechanical system is asymptotically stable [26], [3].

Starting from an equilibrium condition, if the system is asymptotically stable, after the superimposition of an input variation, it will tend to a new equilibrium configuration. If the new equilibrium configuration is sufficiently *near* to the reference one, we can assume that the system can be locally linearised. From the linearization of the object equilibrium equation (2), we obtain:

$$\Delta w + G\Delta\lambda = 0 \tag{14}$$

Let us then consider the hand equilibrium equation (5), the joint torque variation $\delta\tau$ can be expressed as

$$\Delta\tau = J^{\mathrm{T}}\Delta\lambda + K_{J,q}\Delta q + K_{J,u}\Delta u \tag{15}$$

where $K_{J,q} = \frac{\partial J^{\mathrm{T}}\lambda_0}{\partial q}$ and $K_{J,u} = \frac{\partial J^{\mathrm{T}}\lambda_0}{\partial u}$ take into account the variation of hand Jacobian matrix with respect to q and u variations, respectively. Matrices $K_{J,q} \in \Re^{n_q\times n_q}$ and $K_{J,u} \in \Re^{n_q\times n_d}$ are usually referred to as *geometric stiffness matrix* [14]. Their values depend on hand kinematic properties (and, in particular, hand Jacobian derivatives with respect to hand configuration) and on the contact force value in the initial reference configuration, λ_0.

Finally, concerning the relationship between the generalized Lagrangian forces and joint torques, we can express the variation $\delta\sigma$ as follows:

$$\Delta\sigma = S^{\mathrm{T}}\Delta\tau + K_{S,z}\Delta z \tag{16}$$

where $K_{S,z} = \frac{\partial S^{\mathrm{T}}\tau_0}{\partial z} \in \Re^{n_z\times n_z}$ takes into account the variation of S matrix with respect to hand configuration, also in this case its elements are dimensionally a stiffness, and depends on hand kinematics and on the initial reference values of the joint torques, τ_0.

All the main equations of grasping, defined in equations (14), (15), (16), (9), (10), (13), (7), can be grouped in a single linear system:

$$A\Delta\xi = \Delta y \tag{17}$$

with

$$A = \begin{bmatrix} -G & 0 & 0 & 0 & 0 & 0 & 0 \\ J^{\mathrm{T}} & K_{J,u} & -I & K_{J,q} & 0 & 0 & 0 \\ 0 & 0 & S^{\mathrm{T}} & 0 & -I & K_{S,z} & 0 \\ C_c & G^{\mathrm{T}} & 0 & -J & 0 & 0 & 0 \\ 0 & 0 & -I & -K_q & 0 & 0 & K_q \\ 0 & 0 & 0 & 0 & C_z & I & 0 \\ 0 & 0 & 0 & 0 & 0 & S & -I \end{bmatrix} \tag{18}$$

$$\Delta\xi = [\Delta\lambda \;\; \Delta u \;\; \Delta\tau \;\; \Delta q \;\; \Delta\sigma \;\; \Delta z \;\; \Delta q_r]^{\mathrm{T}}$$

$$\Delta y = [\Delta w \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; \Delta z_r \;\; 0]^{\mathrm{T}}$$

The linear system consists of $n_i = n_d + 3n_q + 2n_z + n_l$ equations whose solution represents the solution of the grasp analysis problem for underactuated and compliant grasps. If the matrix of coefficients is not singular, it is possible to find a solution for the unknowns for a given synergy reference variation Δz_r and/or for a given variation of the external load Δw.

A. Grasp compliance evaluation

In the following we evaluate the explicit relationship of the grasp stiffness matrix K relating the variation of object position Δu with an external wrench variation Δw

$$\Delta w = K\Delta u$$

when the references of the actuation is kept constant, i.e. $\Delta z_r = 0$.

For the sake of simplicity we report here the analytical computation of K obtained neglecting the geometric terms $K_{J,q}$ and $K_{J,u}$:

$$K = GK_{c,eq}G^{\mathrm{T}} \tag{19}$$

where

$$K_{c,eq} = \left(K_c^{-1} + JK_q^{-1}J^{\mathrm{T}} + JSK_z^{-1}S^{\mathrm{T}}J^{\mathrm{T}}\right)^{-1} \tag{20}$$

is the equivalent contact stiffness matrix, that takes into account all the system compliance sources. [1]

In the following numerical example we will evaluate grasp stiffness matrix for a simple planar gripper, including the geometrical terms, and we will analyse its dependency on some grasp parameters.

IV. Numerical example

As an example, let us consider a simple gripper like the one shown in Fig. 2. The gripper is planar and has two fingers, each finger is composed of two phalanges with the same lengths: the gripper has then 4 DoFs. Let $J_1, \cdots, J_4$

[1]It is easy to show that the equivalent contact stiffness matrix allows to express the contact force variation $\Delta\lambda$ as:

$$\Delta\lambda = K_{c,eq}\left(JS\Delta z_r - G^{\mathrm{T}}\Delta u\right)$$

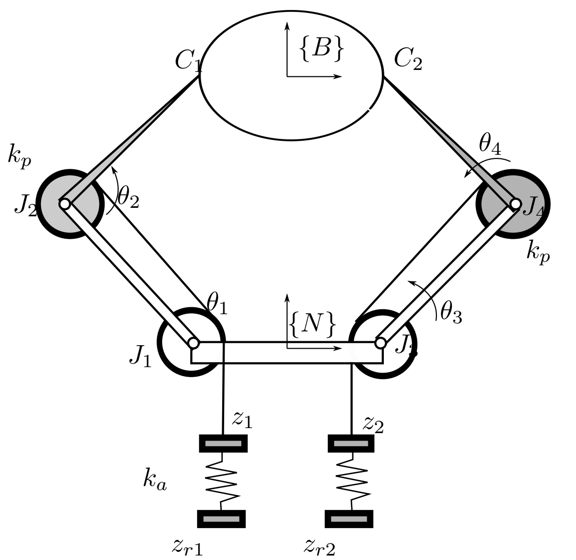

Fig. 2: Example of an underactuated four DoFs gripper, actuated by two elastic tendons, with two passive elastic elements in the joints J_2 and J_4.

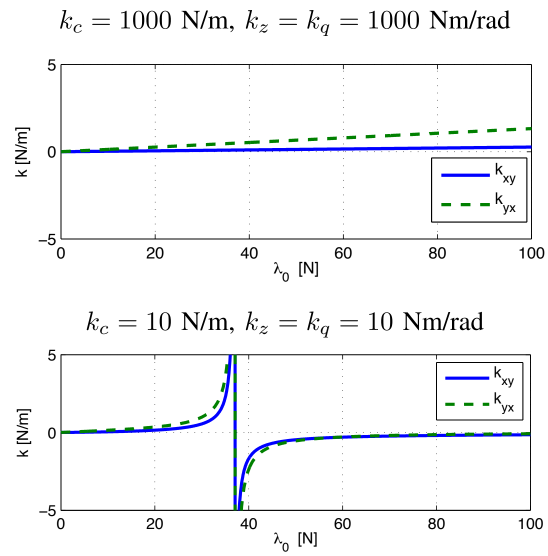

Fig. 3: Linear stiffness terms k_{xy} and k_{yx}, for different system compliance values, and different contact force values.

the joint axis traces on the gripper plane, and let $\theta_1, \cdots, \theta_4$ be the joint angles. The gripper is grasping an object with its fingertips, the contact points are C_1 and C_2, the origin of the local object reference frame is on the mean point of C_1C_2 segment, and the local x axis is parallel to C_1C_2 direction.

The contact model assumed in this test is the hard finger. The object displacement is defined with respect to the base reference system by the vector $u = [u_x \quad u_y \quad \phi]^{\mathrm{T}}$, where ϕ represents the angle between the local and base reference systems abscissae axes.

Indicating with a the length of the finger phalanges, the hand Jacobian matrix is defined as follows

$$J = \begin{bmatrix} J_{1,1} & J_{1,2} & 0 & 0 \\ J_{2,1} & J_{2,2} & 0 & 0 \\ 0 & 0 & J_{3,3} & J_{3,4} \\ 0 & 0 & J_{4,3} & J_{4,4} \end{bmatrix}$$

The grasp matrix is given by

$$G = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & -r & 0 & r \end{bmatrix}$$

where r represents object radius.

The geometric terms, are considered by defining the matrices $K_{J,q}$ and $K_{J,u}$, that can be easily evaluated by calculating the derivatives of $J\lambda_0$ vector w.r.t. q and u components, respectively.

A first numerical analysis was devoted to investigate the effect of geometrical stiffness terms on the total grasp stiffness. We considered a reference configuration in which: $\theta_1 = \frac{3}{4}\pi$ rad, $\theta_2 = -\frac{\pi}{2}$ rad, $\theta_3 = \frac{\pi}{4}$ rad, $\theta_4 = \frac{\pi}{2}$ rad, $w_0 = 0$ N, $\lambda_{01} = [\lambda_0, 0]^{\mathrm{T}}$ N, $\lambda_{02} = [-\lambda_0, 0]^{\mathrm{T}}$ N, where λ_0 value was varied from 0 to 100N, $a = 0.1$ m. Initially the system was considered fully actuated, i.e. $S = I_{4,4}$. The stiffness matrices were $K_c = k_c I_{4,4}$, $K_q = k_q I_{4,4}$, $K_z = k_z I_{4,4}$ where k_c was expressed in N/m, k_q and k_z in Nm/rad, their numerical values were simultaneously varied from 10 to 1000.

Fig. 3 shows the obtained results in terms of grasp stiffness matrix elements. In these numerical simulations, both the stiffness values and the initial contact force values were varied, in order to investigate the weight of geometrical terms in the overall system stiffness. As it could be expected, for high stiffness values, e.g. $k_c = 1000$ N/m, $k_z = k_q = 1000$ Nm/rad, the effect of contact force variation is negligible: the extra-diagonal terms increase linearly as λ_0 increases, and, for high stiffness values, their values are substantially independent from system stiffness values. As the stiffness decreases, e.g. $k_c = 10$ N/m, $k_z = k_q = 10$ Nm/rad, the effect of contact force variation becomes significant and either a discontinuity in the overall grasp stiffness terms appears.

Let us assume that each finger is driven by a tendon as shown in Fig. 2. This actuation system is a simplified illustrative example inspired by the work presented in [10]. The system has still four degrees of freedom, but only two actuators. We can define the Lagrangian variable vector as $z = [z_a \quad z_b \quad q_2 \quad q_4]^{\mathrm{T}}$, in which z_a and z_b represents tendon displacements and are actively controlled, while q_2 and q_4 are passive. From the kinematic analysis of the mechanism it results that

$$S = \begin{bmatrix} \frac{1}{r_1} & 0 & -\frac{r_1}{r_2} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{1}{r_1} & 0 & -\frac{r_1}{r_2} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where r_1 and r_2 are the pulley radii on the joints $J_1 - J_3$ and $J_2 - J_4$, respectively. It is worth to note that in this case S is constant and then $K_{S,z} = 0$. The stiffness matrix K_z is defined as: $K_z = \mathrm{diag}(k_a, k_a, k_p, k_p)$. In this case,

$k_p = 100$ Nm/rad, k_a variable

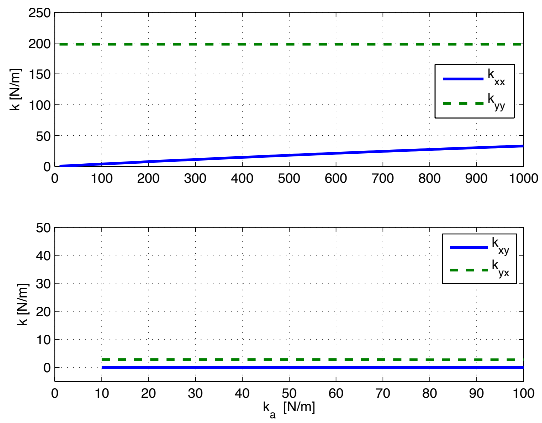

Fig. 4: Linear stiffness terms k_{xx}, k_{yy}, k_{xy}, k_{yx}, for different active and passive stiffness values.

we analysed the translational part of the grasp stiffness matrix varying some stiffness parameters. In particular, first we varied k_p and then k_a keeping all the other parameters constant. The obtained results are shown in Fig. 4.

V. Conclusions

The paper presents a quasi-static model of an underactuated compliant robotic hand grasping an object. The proposed model allows to analyse underactuated hands where some of the DoFs are not actively controlled. The role of compliance is analysed taking into account different sources of compliance. In a quasi-static stetting the grasp analysis has been solved and the grasp stiffness has been evaluated highlighting the terms depending on geometrical parameters, on the system compliance and on the applied force.

The proposed model allows to find a relationship between grasp stiffness and the main structural and control parameters of the hand, then it can be adopted both as an analysis tool, e.g. to investigate the role of geometrical term in the whole grasp stiffness evaluation, and as a design tool, to dimension and optimize hand parameters, e.g. passive joint compliance values.

Future improvements of this study will concern with the compliance design problem. The objective will be the definition of hand structural and control parameters that allows to obtain, in the object space, a suitably defined overall grasp stiffness matrix. The results presented in this paper, and in particular the grasp stiffness matrix, expressed as a function of system stiffness matrices and system structural properties, will be the starting point of this synthesis problem.

References

[1] A. Bicchi and D. Prattichizzo, "Analysis and optimization of tendinous actuation for biomorphically designed robotic systems," *Robotica*, vol. 18, no. 1, pp. 23–31, January 2000.

[2] J. Butterfass, M. Grebenstein, H. Liu, and G. Hirzinger, "DLR-hand II: next generation of a dextrous robot hand," in *Proceedings, IEEE International Conference on Robotics and Automation*, vol. 1, 2001, pp. 109–114.

[3] A. M. Dollar and R. D. Howe, "Joint coupling design of underactuated grippers," in *30th Annual Mechanisms and Robotics Conference*, 2006, pp. 903–911.

[4] L. Birglen, T. Lalibertè, and C. Gosselin, *Underactuated Robotic Hands*, ser. Springer Tracts in Advanced Robotics, B. Siciliano, O. Khatib, and F. Groen, Eds. Springer, 2008, vol. 40.

[5] D. Prattichizzo, M. Malvezzi, and A. Bicchi, "On motion and force controllability of grasping hands with postural synergies," in *Proceedings of Robotics: Science and Systems*, Zaragoza, Spain, June 2010.

[6] M. Santello, M. Flanders, and J. F. Soechting, "Postural hand synergies for tool use," *The Journal of Neuroscience*, vol. 18, no. 23, pp. 10 105–10 115, December 1998.

[7] L. Birglen and C. M. Gosselin, "Kinetostatic analysis of underactuated fingers," *IEEE Trans. on Robotics*, vol. 20, no. 2, pp. 211 – 221, 2004.

[8] S. Krut, V. Bégoc, E. Dombre, and F. Pierrot, "Extension of the form closure property to underactuated hands," *IEEE Trans. on Robotics*, vol. 26, pp. 853–866, 2010.

[9] A. Bicchi and D. Prattichizzo, "Manipulability of cooperating robots with passive joints," in *Proceedings, IEEE International Conference on Robotics and Automation*, 1998, pp. 1038 –1044.

[10] L. U. Odhner and A. Dollar, "Dexterous manipulation with underactuated robotic hands," in *Proceedings, IEEE International Conference on Robotics and Automation*, Shanghai. China, 2011.

[11] S. Krut, "A force isotropic underactuated finger," in *Proceedings, IEEE International Conference on Robotics and Automation*, Barcelona, Spain, 2005.

[12] M. Cutkosky and I. Kao, "Computing and controlling the compliance of a robotic hand," *IEEE Trans. on Robotics*, vol. 5, no. 2, pp. 151–165, 1989.

[13] I. Kao and C. Ngo, "Properties of the grasp stiffness matrix and conservative control strategies," *International Journal of Robotics Research*, vol. 18, no. 2, pp. 159–167, 1999.

[14] A. Pashkevich, K. A., and D. Chablat, "Enhanched stiffness modeling of manipulators with passive joints," *Mechanism and Machine Theory*, no. 46, pp. 662–679, 2011.

[15] S. Chen and I. Kao, "Conservative congruence transformation for joint and cartesian stiffness matrices of robotic hands and fingers," *International Journal of Robotics Research*, vol. 19, no. 9, pp. 835–847, sep 2000.

[16] S. Chen, Y. Li, and I. Kao, "A new theory in stiffness for dextrous manipulation," in *Proceedings, IEEE International Conference on Robotics and Automation*, Seoul, Korea, 2001.

[17] D. Aukes, B. Heyneman, V. Duchaine, and M. R. Cutkosky, "Varying spring preloads to select grasp strategies in an adaptive hand," in *Proceedings, IEEE/RSJ International Conference on Intelligent Robots and Systems*, San Francisco, US, 2011, pp. 1–7.

[18] M. Gabiccini, A. Bicchi, D. Prattichizzo, and M. Malvezzi, "On the role of hand synergies in the optimal choice of grasping forces," *Autonomous Robots*, 2011.

[19] D. Prattichizzo and J. Trinkle, "Grasping," in *Handbook on Robotics*, B. Siciliano and O. Kathib, Eds. Springer, 2008, pp. 671–700.

[20] R. Murray, Z. Li, and S. Sastry, *A mathematical introduction to Robotic Manipulation*, C. Press, Ed., 1994.

[21] M. T. Ciocarlie and P. K. Allen, "On-line interactive dexterous grasping," in *Haptics: Perception, Devices and Scenarios*, ser. LNCS, M. Ferre, Ed., vol. 5024. Springer, 2008, pp. 104–113.

[22] C. Y. Brown and H. H. Asada, "Inter-Finger Coordination and Postural Synergies in Robot Hands via Mechanical Implementation of Principal Components Analysis," in *Proceedings, IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2007, pp. 2877–2882.

[23] M. Ciocarlie, C. Goldfeder, and P. Allen, "Dimensionality reduction for hand-independent dexterous robotic grasping," in *Proceedings, IEEE/RSJ International Conference on Intelligent Robots and Systems*, 2007, pp. 3270–3275.

[24] A. Bicchi, "Force distribution in multiple whole-limb manipulation," in *Proceedings, IEEE International Conference on Robotics and Automation*, Atlanta, 1993, pp. 196–201.

[25] M. Ciocarlie and P. Allen, "A design and analysis tool for underactuated compliant hands," in *Proceedings, IEEE/RSJ International Conference on Intelligent Robots and Systems*, oct. 2009, pp. 5234 –5239.

[26] D. Prattichizzo and A. Bicchi, "Dynamic analysis of mobility and graspability of general manipulation systems," *IEEE Trans. on Robotics and Automation*, vol. 14, no. 2, pp. 241–258, April 1998.