Underactuated Robotic Hands with Synergies

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Grasping vs manipulation dilemma

The simple gripper

- firm grasps
- no manipulation abilities
  - manipulation can be gained with a robotic arm but ...
  - large motions of the arm for small displacements
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- grasp + manipulation
- all joints actuated
- high cost, not robust, control problems
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Trade-off between one and too many actuators? Underactuation: $n_a < n_q$. 
Human hand ? sensori-motor synergies

- simple gripper
- trade-off ?
- fully actuated hand

- Is the human hand a good example to look at ?
- Is the human hand underactuated ?
  Not really, but it works as if it was. Sensori-motor synergies.
Constraints on DoFs: studies from neuroscience

Here human and robotic hands are both modeled with the same mathematical bio-inspired model. We indifferently refer to both robotic and human hand.
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- These constraints do not limit performance rather represent tools allowing the brain to deal with the huge redundancy of sensory and motor apparatuses
- Extensive neuroscientific evidence for the existence of sensorimotor synergies and constraints Babinski, Bernstein, Bizzi, Arbib, Jeannerod, Wolpert, Latash, Flanagan, Soechting, Sperry, ...
- Quantitative work (PCA) dates back a decade only [Santello 98, 02]
Synergistic motions $q(t) = S \dot{z}(t)$

$$S = \begin{bmatrix} \vdots & \vdots & \vdots & \vdots & \ldots & \vdots \end{bmatrix} \in \mathbb{R}^{20 \times 20}$$

The first three components, i.e. columns $S_1$, $S_2$ and $S_3$ account for the 90% of the data.

$$q = S_1 \dot{z}_1 \quad q = S_2 \dot{z}_2 \quad q = S_3 \dot{z}_3$$
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Model underactuated robotic hands inspired by postural synergies.

Hands [Ciocarlie, Allen, Asada, Vassura Melchiorrri, Carrozza, Gosselin, ...]
Grasp and manipulation abilities

**Quasi-static model:**

\[
\begin{align*}
\tau &= J^T \lambda, \\
\mathbf{w} &= -G\lambda.
\end{align*}
\]

where \(G\) is the Grasp Matrix and \(J\) the Hand Jacobian. From the 2nd eq.

\[
\lambda = -G^+ \mathbf{w} + A\xi
\]

The term \(A\xi\) represents the homogeneous solution, when no external load \(\mathbf{w}\) is applied and are usually referred to as *internal forces* (null space of \(G\)).

The contact constraint where \(\Delta u\) is the motion of the object,

\[
\begin{bmatrix}
J & -G^T
\end{bmatrix}
\begin{bmatrix}
\Delta q \\
\Delta u
\end{bmatrix} = 0
\]
Internal force and object motions

- The dimension of the reachable internal force subspace is 2 (grasp)
- The dimension of the reachable motions of the object is 1 (manipulation)
Underactuation, grasp and manipulation

- Reachable force subspace dimension: from 2 to 1
- Reachable object motion dimension: from 1 to 0

Grasp ability (Internal forces) and manipulation ability (object motions) exhibit a dimensional reduction with underactuation.
Underactuation, grasp and manipulation

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Grasp ability (Internal forces) and manipulation ability (object motions) exhibit a dimensional reduction with underactuation.
Underactuation: synthesis and analysis problem

Consider a given kinematic structure. For example this bio-inspired hand.

[Synthesis problem:] how many motors (lower than number of joints) are needed? Which relationships between motors and joints?

[Analysis problem:] For a given underactuated hand (and a given grasp), what are the internal forces and the object motions that the underactuated hand can control?

This is a difficult problem for underactuated hands with general kinematics and grasp configuration.
Defining postural synergies with compliance

- We suppose that the hand is actuated using a number of inputs whose dimension is lower than the number of hand joint and we define it as *synergies*.

- We define the *postural synergies* as a joint displacement aggregation corresponding to a reduced dimension representation of hand movements according to a compliant model of joint torques.

- The reference vector $q_r$ for joint variables is a linear function of postural synergies $z \in \mathbb{R}^{n_z}$ with $n_z \leq n_q$

$$q_r = Sz$$

through the *synergy matrix* $S \in \mathbb{R}^{n_q \times n_z}$ whose columns describes the shapes, or directions, of each synergy in the joint space.
Forces and motions controlled by synergies

From $\Delta z_r$, one gets separate and not combined changes of contact forces $\Delta \lambda$ and object position $\Delta u$

$$
\Delta \lambda = (I - G_K^+ G) KJSY \Delta z_r
$$

$$
\Delta u = \left( GKG^T \right)^{-1} GKJSY \Delta z_r
$$

$$
\Delta z = Y \Delta z_r, \quad \Delta q = XSY \Delta z_r
$$

where $X = (I - C_q J^T (I - G_K^+ G K J)),

K = (C_s + JC_q J^T)^{-1}

and $Y = \left( S^T X^T C_q^{-1} (I - X) SC_z + I \right)^{-1}$

$\Delta u$ and $\Delta \lambda$ cannot be jointly, coordinately, controlled. The dimension of the controlled output $(\Delta u^T, \Delta \lambda^T)^T$ is larger than the dimension of controlled inputs, i.e. the number of synergies.
Reachable internal forces $\Delta \lambda$

The control of internal forces is paramount in controlling the grasp. It allows to steer the contact forces to satisfy the constraints imposed by friction models at the contacts thus guarantying to not loose the contact with the object which would compromise the whole grasp.

The subspace of controllable internal forces by postural synergies:

$$\mathcal{R}(E_s) = \mathcal{R}((I - G_K^+ G)KJSY).$$

All internal forces controllable by synergy actions can be parametrized through a free vector as $E_s \alpha$. 

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Two types of reachable object displacements $\Delta u$

Both of them are reachable controlling synergies but we are interested more in rigid-body motions which can be regarded as low energy motions, in other words they represent the natural way to change the posture of the grasped object.
Rigid-body object motions

- Rigid-body motion controllable by synergies has to be compatible with kinematic contact constraints. Thus a description of this motion can be obtained computing

\[ \text{ker} \left[ \begin{bmatrix} JXS & -G^T \end{bmatrix} \right] \]

- Under the hypothesis that the object motion is not indeterminate, i.e. \( \text{ker}(G^T) \neq 0 \), matrix \( \Gamma \) can be expressed as

\[
\Gamma = \text{ker} \left[ \begin{bmatrix} JXS & -G^T \end{bmatrix} \right] = \begin{bmatrix}
\Gamma_{zr} & \Gamma_{zcs} \\
0 & \Gamma_{ucs}
\end{bmatrix},
\]

- \( \Gamma_{zr} \) is a basis matrix of the subspace of redundant motions \( \text{ker}(JXS) \),
- \( \Gamma_{zcs} \) and \( \Gamma_{ucs} \) are conformal partitions of a complementary basis matrix.
- The image spaces of \( \Gamma_{zcs} \) and \( \Gamma_{ucs} \) consist of coordinated rigid–body motions of the mechanism, for the postural synergy references and the object position and orientation, respectively.
Disjoint internal force and object motion control

\[ \Rightarrow \quad \text{Internal forces } (\mathcal{R}(E_s)) \]

\[ \Rightarrow \quad \text{Non rigid-body motions} \]

\[ \Rightarrow \quad \text{Rigid-body motions } (\mathcal{R}(\Gamma_{ucs})) \]
“Jointly” Internal force and object motion control
Internal force and object motion control

⇒ Internal forces \( \mathcal{R}(E_s) \)

⇒ Non rigid-body motions

⇒ Rigid-body motions \( \mathcal{R}(\Gamma_{ucs}) \)

What about controlling together (jointly and coordinately) both object motions and internal forces (and redundancies) via synergies?
Internal force and object motion control

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* ▷ Internal forces ($\mathcal{R}(E_s)$)

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What about controlling together (jointly and coordinately) both object motions and internal forces (and redundancies) via synergies? Both internal forces and rigid-body object motions.
Main result on force and motion control

- Algebraically, this corresponds to state that for any $\alpha$, $\beta$ and $\gamma$, there always exists a $\Delta z_r$ solving the linear system of equations

$$
\begin{bmatrix}
E_s\alpha \\
\Gamma_{ucs}\beta \\
\Gamma_{zr}\gamma
\end{bmatrix}
= 
\begin{bmatrix}
(I - KG^T(GKG^T)^{-1}G)KJS \\
(GKG^T)^{-1}GKJS \\
I
\end{bmatrix}
\Delta z_r
$$

- Moreover, solution for $\Delta z_r$ is unique and the number of synergies $n_z$ is equal to the sum of the dimensions of the controlled output subspaces:

$$
n_z = \dim(E_s) + \dim(\Gamma_{ucs}) + \dim(\Gamma_{zr})$$
Tripod grasp analysis

- Contact points have been located at the tip of each finger.
- Tripod grasp: thumb, index and middle fingers. The contact points are 3.
- Hard Finger (HF) contact models has considered in this study.

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<th>matrix</th>
<th>dimension</th>
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Tripod example

Dimensions of the controllable internal force, redundancy and allowable movements in case of fully actuated hand and increasing the number of the selected synergies; Case 1: 3 contact points, HF model

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Controllable internal forces and rigid movements

Controllable internal forces

Controllable object motion
Controllable internal forces and rigid movements

Controllable internal forces

Controllable object motion
Conclusions

A framework for underactuated robotic hands based on synergy
Analysis of motion and force controllability in the light of synergies

- How many synergies/actuators have to be involved for a given grasp?
- Which are the contact forces which result to be controllable when acting along
  synergies instead of each single actuator?
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  synergies instead of each single actuator?

Next:
- From analysis to synthesis
- Planning of complex motions with synergies

- European Projects FP7 : THE. “THE Hand Embodied”
- European Projects FP7 : HANDS.DVI “A DeVice-Independent programming and control framework for robotic HANDS”

Thanks for your attention.